Film Cooling by Oblique Slot Injection in High-Speed Laminar Flow

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Wall-cooling effectiveness is investigated for oblique injection of coolant through single or multiple wall slots into a high-speed laminar compressible boundary layer by numerical solutions of the boundary-layer equations. A grid control procedure which maintains a constant flow rate between grid lines is found to be well suited to the present injection calculations wherein the boundary-layer growth in the slot is as much as a hundred-fold and the logitudinal component of the injection velocity is in some cases as large as the freestream velocity. Film-cooling effectiveness is reported for a variety of injection configurations so that the effects of coolant mass flow rate, injection angle, upstream boundary-layer thickness, slot width, and the presence of upstream cooling slots can be investigated. For the coolant mass flow rates considered, normal injection provides better cooling than tangential injection, particularly when frictional heating effects caused by tangential injection become a dominant consideration. However, the excessive boundary-layer growth which accompanies normal injection may reduce aerodynamic performance, thus making inclined injection a desirable compromise.

Nomenclature

C_p = constant pressure specific heat
L = length of leading edge upstream of injection slot
M_∞ = freestream Mach number
$\dot{m}_c$ = coolant mass flow rate
Pr = Prandtl number (taken as 1.0 in present study)
s = slot width
T = temperature
T_{adw} = adiabatic wall temperature upstream of injection slot
T_c = coolant temperature
T_w = wall temperature
T_∞ = freestream temperature
U_j = longitudinal component of injection velocity
U_∞ = freestream velocity
u = longitudinal velocity component
V_j = transverse component of injection velocity
x,y = coordinates parallel and normal to main flow
α = injection angle as shown in Fig. 3.
δ^* = displacement thickness defined in Eq. (6)
η = cooling effectiveness defined in Eq. (5)
ψ = stream function defined in Eq. (2)
ν = kinematic viscosity
ρ = density
τ_w = shearing stress at wall
ω = dimensionless stream function defined in Eq. (4)

Introduction

WALL cooling or reduction in heat transfer by boundary-layer injection is used in engineering applications.[1] Although injection geometry varies, all configurations are included in either of two categories: 1) parallel injection in which a layer of coolant enters beneath the boundary layer through an offset wall; and 2) transverse injection where coolant is blown up into the boundary layer through a slot or holes in the wall. In either of these cases the angle between the coolant flow and the primary flow may vary from tangential to normal. Most previous investigations are confined either to parallel injection which is tangent to the primary flow or to transverse injection which is normal to the primary flow. The present study considers the film-cooling effectiveness of transverse injection at various injection angles through single or multiple wall slots into a compressible boundary layer. Results have been reported for low-speed laminar flow ($U = 50$ m/sec).[2] High-speed laminar flow is reported herein, and work is presently underway to extend the study.

Numerical solutions of the boundary-layer equations are obtained by a finite-difference method which has been extensively tested. Film-cooling effectiveness is presented for a variety of injection configurations so that the effects of freestream Mach number, coolant mass flow, injection angle, boundary-layer thickness, slot width, and the presence of upstream cooling slots can be investigated.

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Numerical Method

A finite-difference method[3] is used to solve the following form of the boundary-layer equations

$$\rho u \frac{\partial u}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial u}{\partial y} = -\frac{dp}{dx} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

$$\rho u \frac{\partial T}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial T}{\partial y} = \frac{u}{C_p} \frac{dp}{dx} + \frac{\partial}{\partial y}\left(\frac{\mu}{Pr} \frac{\partial T}{\partial y}\right) + \frac{\mu}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \quad (1)$$

where the stream function satisfies the requirements

$$\frac{\partial \psi}{\partial y} = \rho u \qquad \frac{\partial \psi}{\partial x} = -\left[\rho_0 v_0 - \int_0^y \frac{\partial}{\partial x}(\rho u)\,dy\right] \quad (2)$$

The solution method is quite general and allows arbitrary specification of equation of state, viscosity model, pressure distribution, wall temperature or heat flux distribution, as well as arbitrary distribution of both transverse and longitudinal velocity components at the injection slot.

A system of finite-difference equations is derived by double integration of the boundary-layer equations over a small control volume and after linearization the difference equations are reduced to the tri-diagonal forms:

$$F_i = A_i \delta u_{i-1} + B_i \delta u_i + C_i \delta u_{i+1}$$

$$\hat{F}_i = \hat{A}_i \delta T_{i-1} + \hat{B}_i \delta T_i + \hat{C}_i \delta T_{i+1} \qquad (3)$$

where δu_i and δT_i are the stepwise changes in velocity and temperature on a typical grid line, and the coefficients are calculated from upstream data which is available before a forward step is taken. The tri-diagonal systems of equations are solved at each forward step of the marching integration process using the algorithm of Thomas.[4] The integration procedes along lines of constant ω as defined by

$$\omega(x,y) = [\psi(x,y) - \psi_0(x)]/\Psi(x) \qquad (4)$$

where ψ is the stream function, ψ_0 and Ψ are a chosen datum and reference respectively, and the grid spacing is expanded or contracted during computation by variation of $\Psi(x)$.

Although the present method traces its origin to that of Patankar and Spalding,[5] major modifications have been undertaken. Test computations have been run for a variety of well-documented flow configurations including wedge flows, flow over a cylinder, Howarth flow, uniform suction and blowing on a flat plate, compressible flow over an adiabatic plate at various Mach numbers, and incompressible flow over a flat plate at various Eckert numbers. To provide a severe test of stability and accuracy, the initial velocity and temperature profiles were taken to be uniform or linear and a very moderate number of grid lines (usually 13) was used. Excellent results were obtained as reported by Nilson and Tsuei.[3] Computation time for compressible flow is approximately 1500 forward steps, roughly 10 typical runs/min on an IBM 370/165.

As a prelude to the oblique injection calculations of the present study, two related test cases, the wake of a flat plate and slot suction, are presented. Although solutions of the Navier Stokes equations for wake flow are available,[6,7] the present method is compared with two other solutions of the boundary-layer equations, Goldstein's series solution and a numerical solution by Plotkin and Flügge-Lotz.[6] The present method uses only 13 interior grid lines, and the most severe case for step size $\Delta x/L = 0.006$ requires four, two, and one iterations, respectively, at the first, second, and third steps, but no iterations thereafter. On the other hand, the boundary-layer solution (not Navier-Stokes solution) by the Plotkin-Flugge-Lotz method uses 39 interior grid lines and requires 8 iterations of each forward step. As indicated in Fig. 1, the center-line velocity of both numerical methods and the velocity profiles of the present method show good agreement with Goldstein's series solution. Figure 2 presents comparative results for a flow with a suction slot in an otherwise impervious wall. All methods[8-10] are in agreement in the suction region and far downstream, whereas the results of Rheinholdt appear to be incorrect immediately downstream of the slot. The velocity profiles of the present method agree with those of Rheinholdt within the accuracy of his graphs throughout the suction slot.

Numerical calculation of film cooling by slot injection is begun at the upstream end of the slot using the similarity velocity and temperature profiles which are appropriate for the adiabatic and impervious leading edge. In the slot the normal and tangential components V_j and U_j, respectively, of the injection velocity and the coolant temperature T_c are specified, whereas downstream of the slot the wall is again adiabatic and impervious. The sudden change in boundary conditions at each end of the slot is introduced linearly in a very short interval as is justified by physical considerations and by a sensitivity study of both the previous slot suction flow and the flows under the most severe injections. To insure adequate grid spacing, another sensitivity study is performed in which it is found that 33 grid lines and 16 steps in the gap are more than adequate.

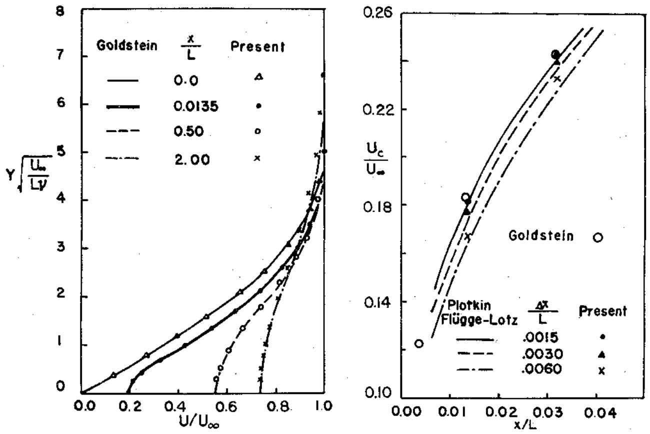

Fig. 1 Velocity profiles and centerline velocity for wake flow.

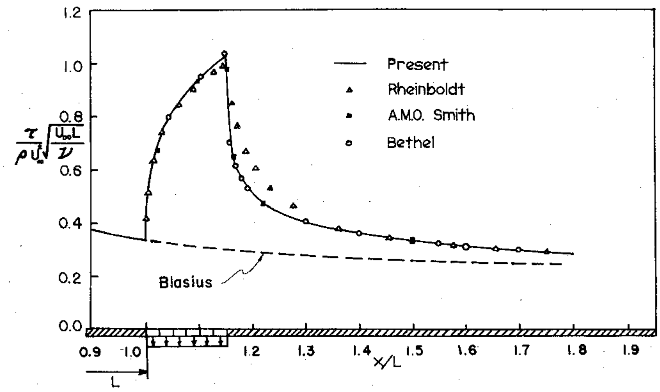

Fig. 2 Comparison of skin friction coefficient for slot suction.

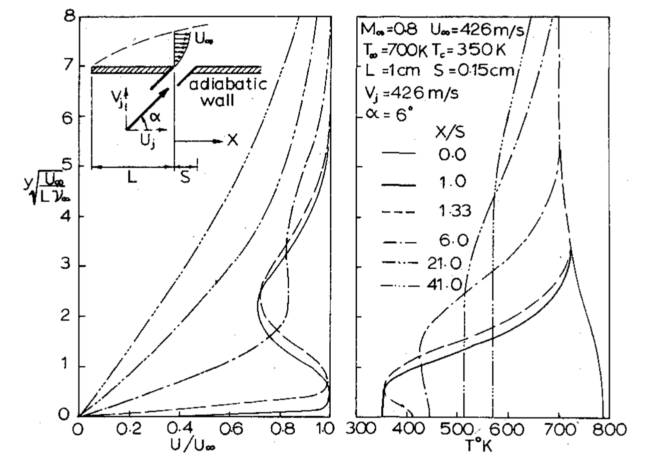

Fig. 3 Velocity and temperature profiles for tangential injection at $M_\infty = 0.8$.

It is noted that the grid system of the present method is extremely well suited to the computation of both normal and tangential injection. In the slot region, the flow rate between the grid lines which bound each grid interval is held fixed. Thus, in normal injection (where the boundary-layer growth in the slot is as much as a hundred-fold), the grid lines spread apart as the velocity in the wall region is retarded, whereas in tangential injection (where the longitudinal velocity at the wall is in some cases as large as the freestream velocity) the grid lines tighten up to provide good coverage of the jet which forms in the wall region.

Results

For the film-cooling results reported here, the geometry is as shown in Fig. 3, the ambient temperature T_∞ is 700 K, the coolant temperature T_c is 350 K, and the freestream Mach number M_∞ is either 0.8 or 2.0. Both the mainstream and the coolant are air at atmospheric pressure, and in view of the large difference between the coolant temperature and the adiabatic wall temperature T_{adw}, variations in density and viscosity are accounted for by the ideal gas law and Sutherland viscosity formula. The cooling effectiveness η as defined by

$$\eta = (T_{adw} - T_w) / (T_{adw} - T_c) \tag{5}$$

is investigated for a variety of parameters. In most cases the leading-edge length L is 1 cm and the slot width s is 0.15 cm. The ratio of the normal component V_j of the injection velocity to the freestream velocity U_∞ is chosen as 0.05 and 0.10 provide variation of the coolant mass flow rate $\dot{m}_c = \rho_c V_j s$ with s held fixed. The ratio of the tangential velocity U_j to the normal velocity V_j at the injection slot is chosen as 10, 1, and 0.1. Thus, the oblique injection angles measured from the mainstream direction are approximately 6°, 45°, and 84° which, respectively, describe the features of tangential, inclined, and normal injections. Typical velocity and temperature profiles for tangential injection are shown in Fig. 3. The influence of injection angle α and the coolant mass flow rate $\dot{m}_c$ on cooling effectiveness is shown in Fig. 4 for $M_\infty = 0.8$ and in Fig. 5 for $M_\infty = 2.0$; and the influence of injection angle and coolant mass flow rate on the boundary-layer displacement thickness as defined by

$$\delta^* = \left[\frac{U_\infty}{L \nu_\infty} \right]^{1/2} \int_0^\infty \frac{\rho}{\rho_\infty} \left(1 - \frac{u}{U_\infty}\right) \mathrm{d}y \tag{6}$$

is shown in Fig. 6 for $M_\infty = 0.8$ where a comparison is made with Crocco's similarity solution for a flat plate with no injection.[11]

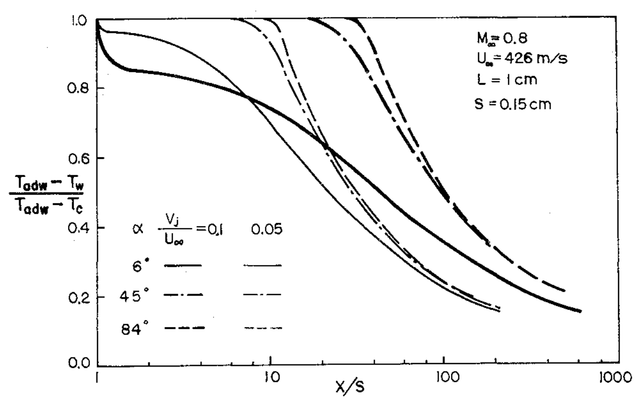

Fig. 4 Comparison of effectiveness for different coolant mass flow rates and injection angles at $M_\infty = 0.8$.

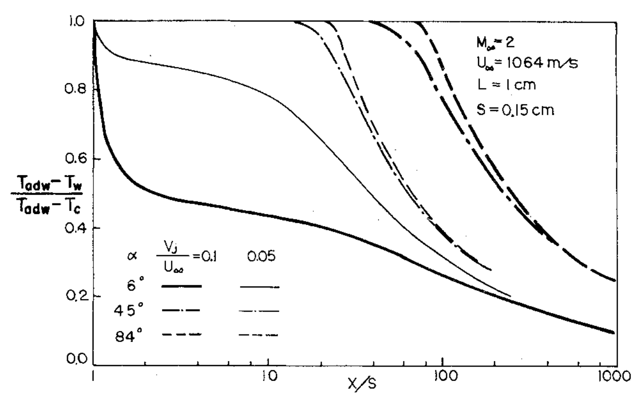

Fig. 5 Comparison of effectiveness for different coolant mass flow rates and injection angles at $M_\infty = 2.0$.

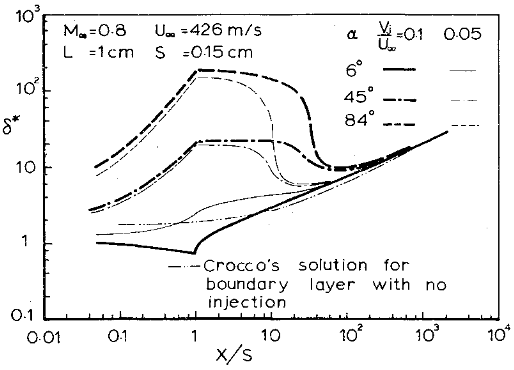

Fig. 6 Displacement thickness for different coolant flow rates and injection angles at $M_\infty = 0.8$.

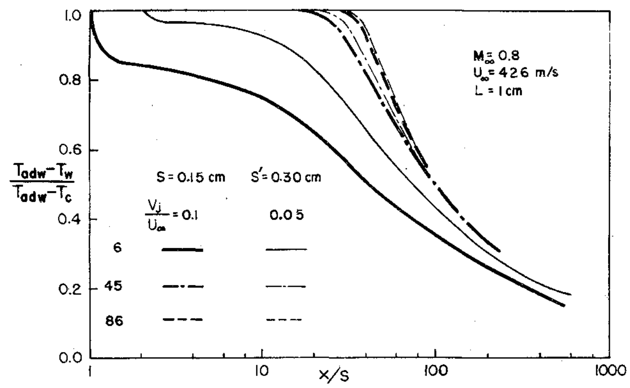

Fig. 7 Comparison of effectiveness for different slot widths at $M_\infty = 0.8$ with coolant flow rate held fixed.

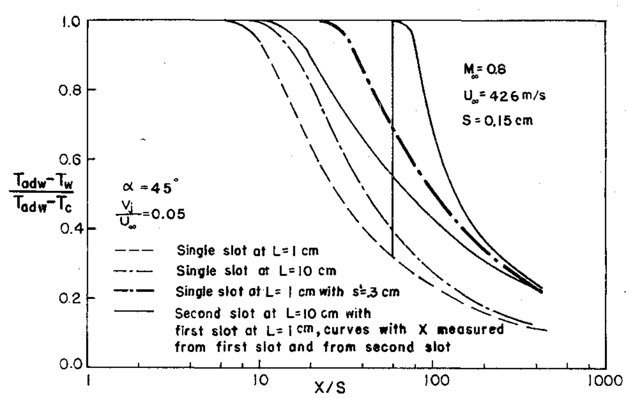

Fig. 8 Comparison of effectiveness of single slot and double slot injections at $M_\infty = 0.8$.

To study the effect of varying s with $\dot{m}_c$ held fixed, a few cases are run with $s' = 0.30$ cm as shown in Fig. 7. To illustrate the influence of upstream boundary-layer thickness, some results are presented in Fig. 8 for $L = 10$ cm. Double-slot cooling is also included in Fig. 8 with the first slot located at $L = 1$ cm followed by a second slot at $L = 10$ cm, both slots being of width $s = 0.15$ cm.

Discussion and Conclusions

An investigation of film cooling in high-speed laminar flow by single or multiple slot injection is presented. Numerical solutions of the boundary-layer equations are obtained with a revised version of the Patankar-Spalding method. Extensive testing has shown the present method to be versatile, accurate, and stable. From the results presented here, several qualitative conclusions are drawn regarding the selection of film-cooling parameters such as slot length s, slot location L, injection angle α, and coolant mass flow $\dot{m}_c$. Although some of the conclusions are similar to those reported for low-speed flow,[2] it is found that, because of frictional heating effects, the freestream Mach number is an important consideration in film cooling design.

1) As indicated in Figs. 4 and 5, normal injection provides much better cooling than tangential injection in the region immediately downstream of the slot. The high effectiveness of normal injection is primarily attributed to the greater thickening effect on the boundary layer (see Fig. 6) as is also observed in low-speed flow. A qualitatively similar heat transfer phenomenon is reported by Sherman et al.[12] in a numerical study of laminar parallel slot injection wherein it is found that for the same coolant flow, increasing slot height provides more effective cooling than increasing jet velocity. Another consideration which becomes more apparent as the Mach number increases is the frictional heating at the end of the slot which reduces the effectiveness of tangential injection. Thus, if high effectiveness, say 0.5, must be maintained, normal injection is preferred and will allow wider spacing of cooling slots. However, in Fig. 6 it is seen that the boundary-layer growth is roughly tenfold for inclined injection and is in excess of a hundredfold for normal injections, whereas the displacement thickness for tangential injection may be less than that of the undisturbed flow because of high longitudinal velocity near the wall. Thus, although normal injection provides greater cooling effectiveness, the attendant boundary-layer growth may induce early separation and reduction of aerodynamic performance.

2) As indicated in Figs. 4 and 5, well downstream of the slot, the cooling effectiveness of tangential injection is nearly equal to that of normal injection. Thus, if only moderate effectiveness, say 0.3, is required, it is preferable to use tangential injection which not only serves the purpose of film cooling but also increases aerodynamic performance. In general, a compromise must be considered in film cooling design.

3) As the Mach number increases, inclined injection becomes very nearly as effective as normal injection, whereas the effectivenss of tangential injection becomes greatly reduced by frictional heating at the end of the slot. Thus, for high Mach numbers, inclined injection appears to be the most advantageous. However, it is the longitudinal component of the injection velocity which controls frictional heating and must be considered in making a choice of injection angle. As seen in Figs. 4 and 5, the frictional heating effect is so severe that the effectiveness of tangential injection may actually be decreased by increasing the coolant flow rate.

4) As the Mach number increases, both normal and inclined injections become more effective because of the higher temperature in the upstream boundary layer which causes increased thickness and lower density thus making it easier to blow the hot fluid away from the wall. However, in tangential injection, the frictional heating at the end of the slot offsets these factors to cause a reduction in effectiveness as the Mach number increases.

5) As indicated in Fig. 7, where coolant mass flow rate is held fixed, a low injection velocity with a wide slot is preferable to a high velocity with a narrow slot. Although the effect is small for normal injection, it is more pronounced for tangential injection.

6) As indicated in Fig. 8, the larger the boundary-layer thickness upstream of the slot, the greater the effectiveness, particularly near the slot region. However, this influence diminished as the injection angle α is decreased.

7) As indicated in Fig. 8, the presence of an upstream slot increases cooling effectiveness. However, the degree of influence is strongly dependent on the spacing between slots, since the increased efficiency is primarily attributed to the reduction of temperature in the boundary layer with increase of boundary-layer thickness being a secondary consideration. As expected, it is also seen in Fig. 8 that a single slot configuration is less effective than a double slot configuration which has the same total coolant flow rate.

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